

# Cosmological acceleration from a gas of strings

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In string gas cosmology, the extra dimensions of the underlying theory are kept at a microscopic scale by a gas of strings. In the matter-dominated era, however, dust pressure can lead to oscillations of the extra dimensions and to acceleration in the three visible dimensions, even with a vanishing cosmological term. We review the resulting oscillating expansion history, that provides an acceptable fit to the observed accelerated expansion of the Universe.

## 1. Introduction

The standard  $\Lambda$  Cold Dark Matter ( $\Lambda$ CDM) model, that reproduces the available experimental observations with remarkable success, posits that the dynamics of the universe is dominated by dark matter (DM) and dark energy [1]. A constant ( $\Lambda$ ) or time-varying (quintessence) vacuum energy could constitute the dark energy explaining the late-time acceleration in the expansion rate of the universe. The dominant component of the matter sector, the DM, does not have appreciable interactions with radiation, and cannot be in the form of ordinary baryonic matter as deduced from considerations of cosmological nucleosynthesis (BBN) together with observations of the anisotropies in the cosmic microwave background (CMB). In addition, studies of the dynamics of galaxy clusters show that the DM particles must also be cold, non-relativistic.

Understanding the nature of these dark components constitutes one of the central challenges in both cosmology and particle physics today. Indeed, all the particles that have ever been observed or artificially created in particle colliders so far fall in the small 5% baryonic component (with the exception of neutrinos, that seem to have too small a mass to play the role of CDM), although there is no lack of viable candidates for constituting the DM particles in extensions of the Standard Model (SM) of particle physics. On the other hand, the observed magnitude of the dark

energy (DE) is orders-of-magnitude smaller than what would be naturally expected.

The particle physics underlying the  $\Lambda$ CDM model is partly described at the quantum level by the SM, which has been successfully verified by all accelerator experiments to date. On the other hand, the dynamics of the expansion is governed in the  $\Lambda$ CDM model by Einstein's theory of the gravitational interactions. Our present knowledge of the gravitational sector at the quantum level, however, is far from complete, and attempts to further our understanding are being actively pursued, particularly in the context of string theory.

Perturbative string theory is most naturally formulated in 9+1 dimensions. The usual way of getting closer to the observed 3+1-dimensional universe is to compactify six spatial dimensions by hand so as to end up with four-dimensional Minkowski space times a six-dimensional compact manifold. The extra dimensions are often taken to be static (indeed, understanding of string theory in time-dependent backgrounds is still quite limited), and compactification is considered not to involve any dynamical evolution. In the search for a static split into large and small spatial dimensions, no explanation has emerged for why there should be three of the former and six of the latter.

From the cosmological point of view, a natural possibility is that the split into three large and six small dimensions arises due to dynamical evo-

lution. String gas cosmology (SGC) (see [2,3] for reviews) is a cosmological scenario motivated by string theory that, unlike in most applications of string theory, treats all spatial dimensions on an equal footing: they are all compactified and start out small, and filled with a hot gas of branes of all allowed dimensionalities. The branes can wind around the tori. The energy of the winding modes increases with expansion due to the tension of the branes, and this resists expansion. As the universe expands and cools down, winding and anti-winding modes annihilate, allowing further expansion. A simple counting argument suggests that  $p$ -branes and their anti-branes cannot find each other to annihilate in more than  $2p+1$  spatial dimensions, so at most  $2p+1$  dimensions can become large. For  $p = 1$ , corresponding to strings, this is three spatial dimensions. Also in contrast to most higher-dimensional proposals, SGC aims to explain not only why some dimensions are hidden, but also why the number of visible dimensions is three (see [4,5] for other proposals along the same lines).

At late times in the universe, the visible spatial dimensions expand, while any compact dimensions which exist must be relatively static. Assuming that the dilaton is stabilised by some other mechanism, the string gas can stabilise the extra dimensions during the radiation-dominated era. However, when the universe becomes matter-dominated, the matter will push the extra dimensions to open up. It was shown in [6] that the gas of strings can still prevent the extra dimensions from growing too large, but they cannot be completely stabilised. There is a competition between the push of matter and the pull of strings. If the number density of the strings is too small, the extra dimensions will grow to macroscopic size. If the strings win, the size of the extra dimensions will undergo damped oscillations around the self-dual radius. The oscillations between expansion and contraction of the extra dimensions induce oscillations in the expansion rate of the large dimensions, which can involve alternating periods of acceleration and deceleration [6]. (This kind of mechanism has also been studied in [7].)

Since the oscillations can start only after the universe becomes matter-dominated, they pro-

vide an in-built mechanism for late-time acceleration in string gas cosmology, one that alleviates the coincidence problem in a manner similar to scaling and tracker fields [8,9].

However, the oscillating expansion history is quite different from the  $\Lambda$ CDM model which is known to be a good fit to the observations. A comparison to observations of type Ia supernovae (SNe Ia), taking into account the BBN constraint on additional radiation degrees of freedom, was performed in [10]. In the following, we review the string gas model [6] and the results, detailed in [10], showing that the oscillating expansion history is not ruled out by the quality of the fit, although it is disfavoured compared to the  $\Lambda$ CDM model.

Our scenario is based on ingredients already present in string gas cosmology and does not require adding new degrees of freedom or turning on new interactions. The late-time evolution of the universe is driven by (classical) gravitational effects. Also, in contrast to the  $\Lambda$ CDM model, there is a fundamental principle that singles out the number of observed spatial dimensions.

## 2. The string gas model

In the string gas model discussed in [6] the spacetime is ten-dimensional, with the metric

$$ds^2 = - dt^2 + a(t)^2 \sum_{i=1}^3 dx^i dx^i + b(t)^2 \sum_{j=1}^6 dx^j dx^j, \quad (1)$$

where  $i = 1 \dots 3$  ( $j = 1 \dots 6$ ) labels the visible (extra) dimensions. All spatial dimensions are taken to be toroidal, and  $b = 1$  corresponds to the self-dual radius given by the string length  $l_s \equiv \sqrt{\alpha'}$ .

We assume that the dilaton has been stabilised in a way that leaves the equation of motion of the metric unconstrained, so that it reduces to the Einstein equation,  $G_{\mu\nu} = \kappa^2 T_{\mu\nu}$ .  $\kappa^2$  is the 10-dimensional gravitational coupling, and we take the cosmological constant to be zero.

We do not consider additional covariantly conserved non-trivial tensors, besides Einstein's, that

can be constructed from the metric and its first and second derivatives in more than four dimensions (e.g. the Gauss-Bonnet term). These higher order curvature terms can be of importance in the early universe, and lead to inflation when there are more than three spatial dimensions. Inflation terminates if the extra dimensions are stabilised so that at most three dimensions are free to expand. This relates graceful exit to the number of large dimensions [11]. This scenario, however, is not realised in the SGC context: in an inflating space the string gas will be diluted, and space isotropizes, with all dimensions growing large.

Given the symmetries of the metric (1), the energy-momentum tensor has the form

$$T^\mu_\nu = \text{diag}(-\rho(t), \overbrace{p(t), \dots, p(t)}^3, \overbrace{P(t), \dots, P(t)}^6), \quad (2)$$

where  $p$  and  $P$  are the pressure in the visible and the extra dimensions, respectively.

### 2.1. The matter content.

In addition to ordinary four-dimensional radiation ( $\gamma$ ) and pressureless matter ( $m$ ), we have a gas of massless strings ( $s$ ) with winding and momentum modes in the extra dimensions and momentum modes in the visible ones.

Assuming that all strings have the same initial momentum in the visible directions,  $M$ , the energy-momentum tensor [6] depends on four parameters: the scale  $M$  and the energy densities  $\rho_{\gamma, in}$ ,  $\rho_{m, in}$  and  $\rho_{s, in}$ . The evolution of the system is determined by the two dimensionless combinations:

$$r \equiv M^{-1} \frac{\rho_{\gamma, in}}{\rho_{m, in}}, \quad f_s \equiv \frac{\rho_{s, in}}{\rho_{\gamma, in}}. \quad (3)$$

Rescaling  $a \rightarrow Ma$ , the total energy density reads

$$\rho = \rho_{m, in} M^{-3} a^{-3} b^{-6} \left( 1 + r a^{-1} + r f_s \sqrt{a^{-2} + b^{-2} + b^2 - 2} \right), \quad (4)$$

and the pressures can be written accordingly [10].

### 2.2. Oscillations and late-time acceleration.

The dynamical effects of the gas of strings can be read from the last term in (4). The string gas

behaves like a scaling solution [8] in the radiation-dominated era and like a tracker solution [9] in the matter-dominated era [6]. The value  $b = 1$  is an attractor point: as long as the initial value of  $b$  is not too large ( $b < \sqrt{2}$  is a necessary condition),  $b$  will rapidly evolve to unity, and the extra dimensions are stable. Then the energy density of the string gas behaves exactly like radiation. When the universe becomes matter-dominated, the string gas starts tracking the matter as the extra dimensions expand. When the extra dimensions are pulled back and contracted by the strings, the visible dimensions start oscillating between deceleration and acceleration. (If the string gas is too weak to prevent the extra dimensions from opening up, they will grow without bound, and there will be no acceleration in the visible dimensions. We are not interested in this possibility.)

Let us stress that, in our scenario, the late-time acceleration does not require vacuum energy (indeed, it would rapidly decompactify the extra dimensions [6]). Since the departure from matter-dominated 4D behaviour is due to both the extra dimensions and the string gas, the apparent dark energy does not obey a simple equation of state. It is noteworthy that the deceleration parameter can dip below the de Sitter value  $-1$ . Such rapid acceleration is usually associated with violation of the null energy condition, i.e. equations of state more negative than  $-1$ . Interestingly, the violation of the null energy condition by the strings makes it possible to have acceleration even when the energy density of the universe is dominated by ordinary matter, with  $\Omega_{tot} \approx \Omega_m < 1$ . However, this does not imply spatial curvature, since the correspondence between spatial flatness and critical density is broken by the extra-dimensional terms in the Hubble law.

Deep in the radiation-dominated era (in particular, during BBN), the energy density of the string gas evolves like radiation, and contributes to the total energy density a fraction  $\Omega_{s, in} = f_s / (1 + f_s)$ , given that the contribution of matter is negligible and  $b = 1$  in the radiation-dominated era. The string fraction  $f_s$  is related to the effective number of additional neutrino species  $\Delta N_\nu$  by  $f_s = 7\Delta N_\nu / 43$  [12]. Allowing for a large elec-

tron neutrino chemical potential, from BBN we have  $\Delta N_\nu \leq 4.1$ , which translates into  $f_s \leq 0.7$ , or  $\Omega_{s,in} \leq 0.4$  [13]. The bound depends on the assumption that the gravitational coupling during BBN is the same as today, which is not necessarily true in the string gas model, since  $G_N \propto b^{-6}$ . If  $b < 1$  today, the gravitational coupling at BBN is reduced relative to the present value, so there is more room for new degrees of freedom. However, generally  $b$  dips below unity only very slightly, and typically  $b > 1$  today, so taking this into account would make the constraints tighter. A requirement for the string gas being able to keep the extra dimensions small is  $rf_s > 3/2$  [6]. There are no other constraints on  $r$ , since it depends on  $M$ , the initial momentum of the strings in the visible directions, on which there is no limit.

### 3. Constraints from SNe Ia

The fact that a matter-dominated period followed by accelerated expansion without decompactification is possible may be seen as a step towards developing SGC into a realistic model of the universe at all eras. However, it is not clear whether the late-time acceleration produced by this mechanism can be in agreement with observations. A detailed study of the parameter space of the model and a comparison to different cosmological datasets was undertaken in [10]. We summarise, in the following, the main results of that analysis.

Two important sets of observations which depend only on the background are luminosity distances of SNe Ia and the primordial abundance of light elements. The Union dataset [14] is the newest and most comprehensive collection of SNe Ia observations, but it has been analysed with the assumption that the  $\Lambda$ CDM model is correct. To check for any potential bias against models which are significantly different from  $\Lambda$ CDM, like the string gas model, it is convenient to also use the ESSENCE SNIa dataset [15].

In comparing the curve of measured luminosity distances *vs.* redshift of SNe Ia, we should keep in mind that the metric (1) does not have the FRW form. The usual expression for the luminosity distance does not hold and the general expression

given in [16] should be used.

The results of a scan in the  $(r, f_s)$  are shown in Figure 1. The complicated  $\chi^2$  contours are not an artifact of the analysis. To obtain enough acceleration in the visible dimensions at sufficiently late times, the present day has to be in a specific location, just after the rise of one of the first few oscillations. Also, in order to have strong acceleration, the extra dimensions have to expand almost to the point of not turning back, and then contract rapidly. If the extra dimensions were to expand slightly more, they would not turn around, and there would be no acceleration. Therefore the best fits are obtained on the border of very poor fits

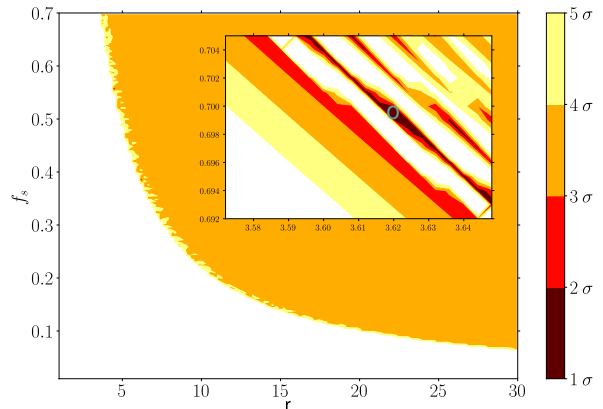


Figure 1. Confidence level contours in the  $(r, f_s)$ -plane for the Union dataset. The best-fit model is marked with a circle.

For the Union dataset, the  $\chi^2$  for the best-fit string gas model without the BBN constraint is 9.3 points worse than for the  $\Lambda$ CDM model, and 21.5 points worse when the BBN constraint is taken into account [10].

In Figure 2, we plot some quantities for the best-fit model to the Union dataset (with the BBN constraint included). The energy density of the string gas is completely subdominant at late times,  $\Omega_{s0} = 0.02$ . However, the string gas

can still have a large impact on the dynamics, because its energy-momentum tensor violates the null energy condition. When the expansion is faster than in the Einstein-de Sitter case, the matter density parameter  $\Omega_m \equiv \kappa^2 \rho_m / (3H_a^2)$  is smaller than unity, and in principle it could be in the observationally allowed range  $\Omega_{m0} \approx 0.2-0.3$  today. However, for the best-fit model we have  $\Omega_{m0} = 0.73$ , far too large.

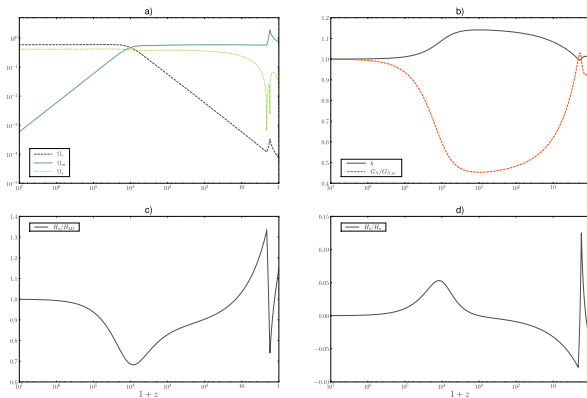


Figure 2. a) Density parameters  $\Omega_i \equiv \kappa^2 \rho_i / (3H_a^2)$ , b) size of the extra dimensions and Newton's constant, c) expansion rate of the large dimensions ( $H_{4D}$  is the Hubble parameter in the usual four-dimensional case) and d) expansion rate of the extra dimensions, for the best-fit model to the Union data, with the BBN constraint.

In Figure 2 b) we show the scale factor of the extra dimensions  $b$  and the four-dimensional gravitational coupling  $G_N \propto b^{-6}$ . The difference between  $b$  at BBN and today is small, and well within the observational limits discussed in [6]. However,  $b$  deviates noticeably from unity at last scattering. This is a generic feature of the string gas model, because last scattering is soon after the matter-radiation equality, when the extra dimensions start opening up. This prediction could provide a stringent constraint. However, quoted

limits on the variation of  $G_N$  (or on new radiation degrees of freedom) from the CMB and other non-BBN probes are model-dependent, and rely on perturbation theory. (Note that the string gas does not behave like radiation at last scattering.)

In Figure 2 c) we show the expansion rate of the visible dimensions  $H_a$  relative to what it would be without the extra dimensions and the string gas, denoted by  $H_{4D}$ . Comparing to the plot of  $H_b/H_a$  in Figure 2 d), we see how acceleration in the visible dimensions correlates with contraction of the extra dimensions. The Hubble parameter today in the model is somewhat low, which is related to the large value of  $\Omega_{m0}$ . In order to get enough acceleration in the recent past, it seems that the extra dimensions must have recently collapsed, so  $b \approx 1$  today. The value  $\Omega_m = 0.3$ , for example, then requires  $H_a/H_{4D} = 1.8$ . The maximum value of  $H_a/H_{4D}$  in the best-fit model is only 1.3, and the value today is 1.1. Without the BBN constraint, the situation would be better, with higher values of  $H_a/H_{4D}$ .

The quantity  $H_a/H_{4D}$  also gives the relation between the age of the universe and the present value of the Hubble parameter, since  $H_a/H_{4D} = 3H_a t/2$  at late times. A model-independent observational constraint on the age of the universe is given by the ages of globular clusters [17], which lead to the lower limit  $t_0 \geq 11.2$  Gyr at 95% C.L. and a best-fit age of  $t_0 = 13.4$  Gyr. The best model-independent measure of the current value of the Hubble parameter comes from the Hubble Key Project [18]. The result is sensitive to the treatment of Cepheids, and two different analyses yield  $H_{a0} = 0.73 \pm 0.06$  km/s/Mpc and  $H_{a0} = 0.62 \pm 0.05$  km/s/Mpc ( $1\sigma$  limits). Taking the best-fit value for  $t_0$  and the mean values for  $H_{a0}$  gives  $H_a/H_{4D} = 1.5$  or  $H_a/H_{4D} = 1.27$ , respectively. The value in the best-fit model is too low, but not drastically so, taking into account the uncertainties in  $t_0$  and  $H_{a0}$ .

#### 4. Discussion and conclusion.

While the string gas model does not fit the SNIa data as well as the  $\Lambda$ CDM model, the fit is not decisively worse when only the SN data are considered, without the BBN constraint. Nev-

ertheless, even the lower goodness-of-fit corresponds to a probability of 15% for the Union data, which is not enough to rule out the model. (For comparison, the goodness-of-fit of the  $\Lambda$ CDM model to the first-year WMAP TT data was 3% [19], and this was considered strong support for the model.)

As discussed in [6], the energy-momentum tensor for the string gas is expected to be more complex than (4), which assumes that all strings have the same momentum  $MI_s/a$  in the visible dimensions. Since the evolution is very sensitive to the parameters of the string gas, a more realistic distribution of strings with different momenta, will lead to quantitatively slightly different oscillations. To explore this possibility, we would have to know the distribution of string momenta, which depends on how the string gas was created in the early universe and whether it has thermalised.

The string gas cosmology context aside, this provides an interesting demonstration of how a model with an expansion history radically different from  $\Lambda$ CDM, but which still provides a good fit to the supernova data. In this context, it may be interesting that the Hubble parameter inferred from observations of the ages of passively evolving galaxies shows oscillations [20], though it is premature to draw strong conclusions from the data.

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